

Addendum to: Quasiparticle random phase approximation uncertainties and their correlations in the analysis of $0\nu\beta\beta$ decay

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In a previous article [Phys. Rev. D **79**, 053001 (2009)] we estimated the correlated uncertainties associated to the nuclear matrix elements (NME) of neutrinoless double beta decay ($0\nu\beta\beta$) within the quasiparticle random phase approximation (QRPA). Such estimates encompass recent independent calculations of NMEs, and can thus still provide a fair representation of the nuclear model uncertainties. In this context, we compare the claim of $0\nu\beta\beta$ decay in ^{76}Ge with recent negative results in ^{136}Xe and in other nuclei, and we infer the lifetime ranges allowed or excluded at 90% C.L. We also highlight some issues that should be addressed in order to properly compare and combine results coming from different $0\nu\beta\beta$ candidate nuclei.

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I. INTRODUCTION

In a previous paper [1] we presented the results of a systematic evaluation of nuclear matrix elements (NME) and of their correlated uncertainties for the neutrinoless double beta decay process ($0\nu\beta\beta$) in different nuclei, within the quasiparticle random phase approximation (QRPA) and the standard framework of light Majorana neutrinos with effective mass $m_{\beta\beta}$. In particular, in [1] we discussed in the joint statistical distribution of the NME values $|M'_i|$ which govern, together with the phase space G_i , the decay half life T_i in the i -th candidate nucleus,

$$T_i^{-1} = G_i |M'_i|^2 m_{\beta\beta}^2, \quad (1)$$

with i spanning the set

$$i = ^{76}\text{Ge}, ^{82}\text{Se}, ^{96}\text{Zr}, ^{100}\text{Mo}, ^{116}\text{Cd}, ^{128}\text{Te}, ^{130}\text{Te}, ^{136}\text{Xe}. \quad (2)$$

We emphasized that the correlations among the NME uncertainties are sizable and play a relevant role in comparing $0\nu\beta\beta$ data from different nuclei, including the ^{76}Ge decay events claimed by Klapdor *et al.* in [2, 3].

Recently, important new limits on the ^{136}Xe half life have been obtained by the experiments EXO-200 [4] and KamLAND-Zen [5], which have reached, for various choices of NME calculations, a 90% C.L. sensitivity to $m_{\beta\beta}$ largely overlapping with the $m_{\beta\beta}$ range favored by the ^{76}Ge claim [4, 5]. These advances have prompted us to use the results in [1] to perform a systematic comparison of ^{136}Xe half-life limits with those obtained in other nuclei and with the ^{76}Ge claim. The comparison is worked out in detail in the next Section, in terms of half-life ranges allowed or excluded at 90% C.L. Once more, NME covariances are shown to play a relevant role in the $0\nu\beta\beta$ phenomenology. We also point out that, in order to fully exploit the implications of upcoming $0\nu\beta\beta$ results, it is generally advisable to discuss in some detail the probability distributions of both the experimental half lives and the theoretical NMEs.

We remark that, in this Addendum to [1], we adopt the same NME and covariances as computed therein. A systematic update of [1] would require extensive QRPA calculations of hundreds of NME, which are left to a future study. However, as we argue in the Appendix, the uncertainties in [1] are conservative enough to embrace recent NME calculations, using either the QRPA or independent theoretical frameworks. We conclude that the NME estimates in [1] still provide a fair representation of the current spread of theoretical $0\nu\beta\beta$ calculations.

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II. COMPARISON OF THE $0\nu\beta\beta$ CLAIM WITH RECENT RESULTS

In this section we briefly review, for the sake of completeness, the notation and conventions used in [1] and the implications of the $0\nu\beta\beta$ claim in ^{76}Ge for different nuclei. Then we compare such implications with recent experimental data, most notably from ^{136}Xe in EXO-200 and KamLAND-Zen, and discuss the regions allowed or excluded at 90% C.L. We remind that gaussian uncertainties at n standard deviations on a given parameter correspond to projections of $\Delta\chi^2 = n^2$ regions on that parameter, and that 90% C.L. uncertainties correspond to $n = 1.64$.

A. Notation, conventions, and implications of ^{76}Ge $0\nu\beta\beta$ claim

As in [1], we linearize Eq. (1) as

$$\tau_i = \gamma_i - 2\eta_i - 2\mu, \quad (3)$$

by taking logarithms of the relevant $0\nu\beta\beta$ quantities in appropriate units:

$$\tau_i = \log_{10}(T_i/\text{y}), \quad (4)$$

$$-\gamma_i = \log_{10}[G_i/(\text{y}^{-1}\text{eV}^{-2})], \quad (5)$$

$$\eta_i = \log_{10}|M'_i|, \quad (6)$$

$$\mu = \log_{10}(m_{\beta\beta}/\text{eV}). \quad (7)$$

The NME central values with their one-standard-deviation errors are denoted as

$$\eta_i = \eta_i^0 \pm \sigma_i, \quad (8)$$

where the σ_i are positively correlated through a matrix ρ_{ij} . Table I in [1] reports the numerical values of γ_i , η_i^0 , σ_i , and ρ_{ij} , which are adopted hereafter.

Let us assume that $0\nu\beta\beta$ decay has been experimentally observed in $i = ^{76}\text{Ge}$ as claimed by Klapdor *et al.* [2, 3], with (logarithmic) half life given at $\pm 1\sigma$ as [1]:

$$\tau_i = \tau_i^0 \pm s_i \quad (9)$$

$$= 25.355 \pm 0.072 \text{ (} i = ^{76}\text{Ge) } . \quad (10)$$

Then, Eq. (3) predicts the following half life in a different nucleus $j \neq i$ [1]

$$\tau_j = \tau_j^0 \pm s_j, \quad (11)$$

where

$$\tau_j^0 = \tau_i^0 + (\gamma_j - \gamma_i) - 2(\eta_j^0 - \eta_i^0), \quad (12)$$

and

$$s_j^2 = s_i^2 + 4(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j), \quad (13)$$

the (s_i, s_j) correlation being given by [1]

$$r_{ij} = \frac{s_i}{s_j} \text{ (} i \neq j \text{) } . \quad (14)$$

We add here that, for two nuclei j and k different from $i = ^{76}\text{Ge}$, the (s_j, s_k) correlation is given by

$$r_{jk} = \frac{s_j^2 + s_k^2 - 4(\sigma_j^2 + \sigma_k^2 - 2\rho_{jk}\sigma_j\sigma_k)}{2s_js_k} \text{ (} j \neq i \neq k \text{) } , \quad (15)$$

see also the Appendix.

TABLE I: Best current limits on half-lives at 90% C.L. ($T_j > T_j^{90}$ and $\tau_j > \tau_j^{90}$) for different nuclei j .

j	T_j^{90}/y	τ_j^{90}	Experiment	Ref.
^{76}Ge	1.6×10^{25}	25.204	IGEX	[6]
^{82}Se	3.6×10^{23}	23.556	NEMO-3	[7]
^{96}Zr	9.2×10^{21}	21.964	NEMO-3	[7]
^{100}Mo	1.1×10^{24}	24.041	NEMO-3	[7]
^{116}Cd	1.7×10^{23}	23.230	Solotvina	[8]
^{128}Te	7.7×10^{24}	24.886	Geochem.	[9]
^{130}Te	2.8×10^{24}	24.447	CUORICINO	[10]
^{136}Xe	3.4×10^{25}	25.531	EXO \oplus KL-Zen	[4, 5]

B. Applications and comparison with recent data

Except for the claim in [2, 3], all other $0\nu\beta\beta$ experiments report negative results to date. Table I shows the current best limits at 90% C.L. on the $0\nu\beta\beta$ half life in different nuclei j (i.e., T_j^{90} and its logarithm τ_j^{90}). Particularly important are the recent limits on ^{136}Xe coming from EXO-200 ($T^{90} = 1.6 \times 10^{25}$ y) [4] and from KamLAND-Zen ($T^{90} = 1.9 \times 10^{25}$ y). A statistical combination of the negative results in [4, 5] is attempted in Ref. [5], where the combined “EXO \oplus KL-Zen” limit $T^{90}(^{136}\text{Xe}) = 3.4 \times 10^{25}$ y is quoted, as reported in Table I and adopted hereafter.

Figure 1 shows the limits reported in Table I (one-sided bands), together with the 90% C.L. ranges implied by the ^{76}Ge claim, as derived from Eqs. (10)–(13) with errors inflated by $\times 1.64$. It can be seen that, for the first time, there is a significant overlap between the half-life limit in one nucleus (^{136}Xe) and the corresponding region favored by Klapdor’s claim for a given set of NME, as also emphasized in the experimental papers [4, 5]. This situation should be contrasted with the analogous Fig. 5 in [1], where no overlap emerged.

In principle, the next logical step should be a combination of positive and negative $0\nu\beta\beta$ results—within the adopted set of NME and their covariances—in order to evaluate the statistical consistency of the data (test of hypothesis) and to identify the range of $m_{\beta\beta}$ consistent with all the results (parameter estimation). This task would require the detailed knowledge of the probability distribution functions, not only for the NME (as attempted in [1]), but also for the ^{76}Ge and ^{136}Xe half lives. However, the half-life likelihoods have not been published in the original papers [2, 3] and [4, 5]. Reproducing or simulating the original data analyses is difficult, especially for the claimed signal in ^{76}Ge , which involved a dedicated pulse-shape discrimination [3]. It would be desirable that future $0\nu\beta\beta$ results are published also in terms of likelihood or χ^2 functions of the half-life, and not only in terms of specific bounds, say, at 90% C.L. Concerning theoretical uncertainties, we also note that after [1] there has been no other independent study of NME error correlations from the viewpoint of different nuclear models. Therefore, we think that the conditions for a quantitative combination or a “global fit” of positive and negative $0\nu\beta\beta$ results are currently not warranted.

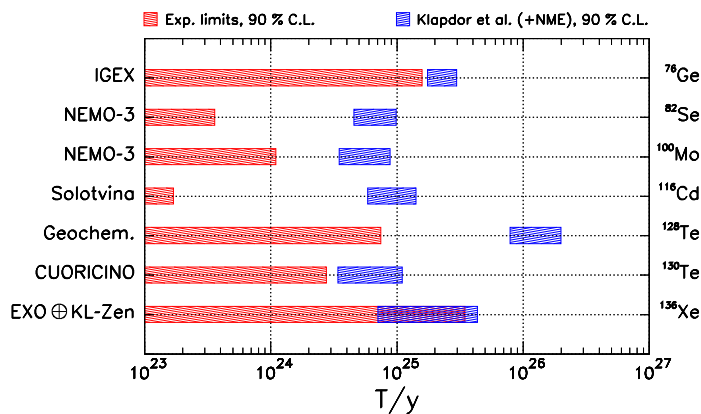


FIG. 1: Range of half lives T_i preferred at 90% C.L. by the $0\nu\beta\beta$ claim of [3], compared with the 90% exclusion limits placed by other experiments. The comparison involves the NME and their errors, as well as their correlations. Note the overlap of favored and disfavored ranges for ^{136}Xe . This figure updates Fig. 5 of [1].

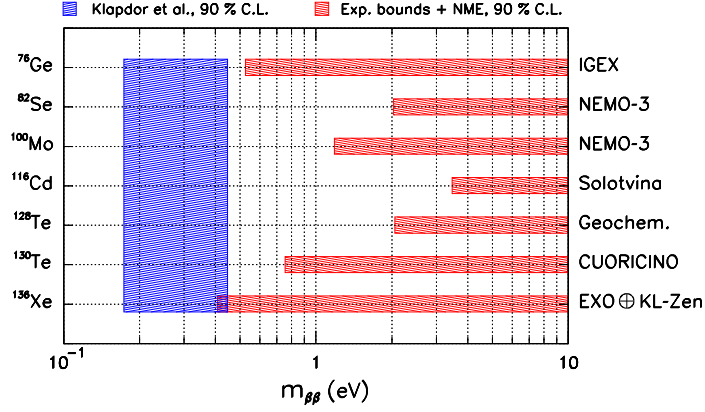


FIG. 2: Range of $m_{\beta\beta}$ allowed by the $0\nu\beta\beta$ claim of [3], compared with the limits placed by other experiments (all at 90% C.L.). This figure updates Fig. 3 of [1].

The above issues emerge, e.g., when one tries to translate the numbers in Table I and Eq. (10) in terms of 90% C.L. limits on $m_{\beta\beta}$ via Eq. (3). In the absence of the experimental likelihood functions for the half lives, the combination of one-sided experimental limits (τ^{90}) with two-sided theoretical errors ($\pm 1.64\sigma_i$) is not obvious. Conservatively, one may combine linearly the experimental and theoretical ranges at 90% C.L. as proposed in [1], at the price of losing statistical power. Figure 2 shows the results of such combination, in terms of favored and disfavored ranges of $m_{\beta\beta}$. It can be noticed that the ^{136}Xe limits overlap with the range favored by the ^{76}Ge claim, but not as much as in Fig. 1, signaling the loss of statistical information. Therefore, we prefer to show the following results directly in terms of the observable half lives T_i and not via $m_{\beta\beta}$.

Figure 3 shows the application of Eqs. (10)–(14) in the plane charted by the half lives (T_i, T_j) for $i = ^{76}\text{Ge}$ and $j = ^{136}\text{Xe}$, at 90% C.L. The horizontal band corresponds to Klapdor’s claim, while the slanted band represents the theoretical range at the $\pm 1.64\sigma$ level [1]. Their combination provides the allowed ellipse, which is however largely disfavored by the one-sided limit placed by EXO \oplus KL-Zen (vertical bound). The surviving ellipse segment corresponds to $T(^{76}\text{Ge}) \simeq 2.0\text{--}2.9 \times 10^{25}$ y and $T(^{136}\text{Xe}) \simeq 3.4\text{--}4.3 \times 10^{25}$ y. The upper ends of such ranges set the limits required to test Klapdor’s claim at 90% C.L. Such an exercise could be repeated at higher C.L., if the corresponding experimental limits or the half-life likelihood were also published; this will become important in the future, since the $\sim 6\sigma$ signal claimed by Klapdor *et al.* [2, 3] should be tested at C.L. definitely higher than 90%.

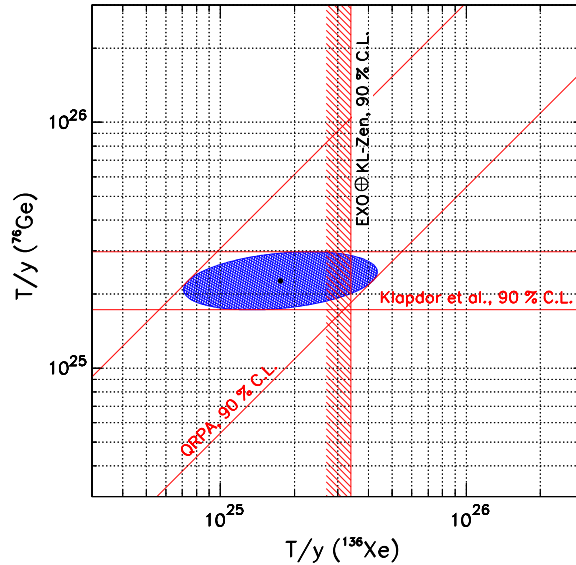


FIG. 3: Theoretical and experimental constraints in the plane charted by the $0\nu\beta\beta$ half-lives of ^{76}Ge and ^{136}Xe . Horizontal band: range preferred by the $0\nu\beta\beta$ claim of [3]. Slanted band: constraint placed by our QRPA estimates [1]. The combination provides the shaded ellipse, whose projection on the abscissa gives the range preferred at 90% C.L. for the ^{136}Xe half life. This range is largely disfavored by the combined EXO \oplus KL-Zen results [4, 5] (vertical one-sided limit).

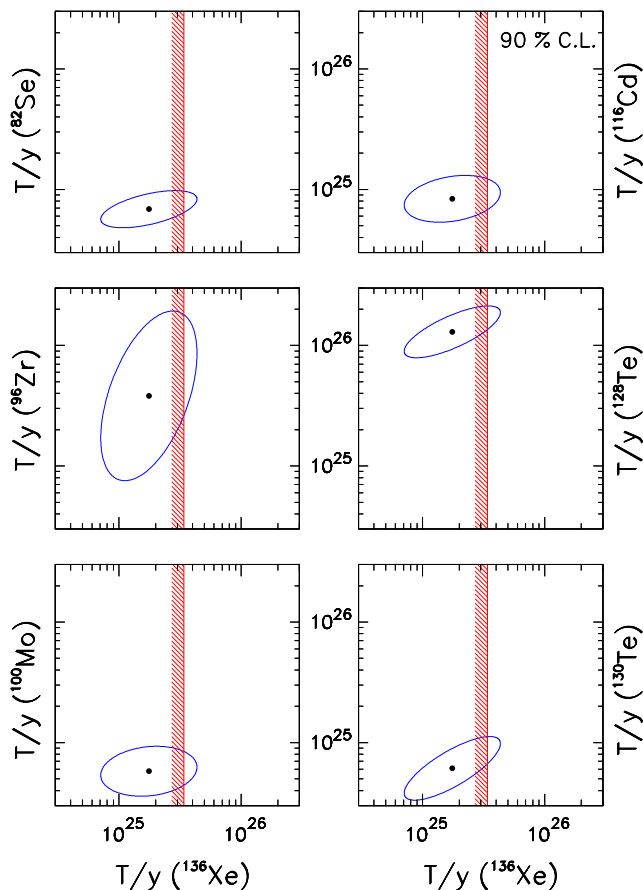


FIG. 4: Allowed regions (ellipses) as derived from Klapdor’s claim [3] and the NME of [1], in the plane charted by the half lives of ^{136}Xe and each of the six nuclei ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{128}Te , and ^{130}Te . A large fraction of each ellipse is excluded by the combined EXO \oplus KL-Zen results [4, 5] (vertical one-sided limit). All bounds are at 90% C.L. on one variable.

Figure 4 shows the application of Eqs. (13) and (15) in the six planes charted by the half lives (T_j, T_k) for $j = ^{136}\text{Xe}$ and $k \neq ^{136}\text{Xe}, ^{76}\text{Ge}$. In each panel, the ellipse represents the region favored by Klapdor’s claim within the adopted NME and their covariances, to be compared with the region excluded by EXO \oplus KL-Zen at the same C.L. (vertical bound). As a consequence of the positive correlation, the ellipse segment not excluded by the ^{136}Xe limit corresponds, for each k -th nucleus, to the higher end of the half-life range at 90% C.L. For instance, in the case of ^{130}Te (lower right panel in Fig. 4), the current limit should be pushed from 2.8×10^{24} y up to $\sim 0.7\text{--}1.1 \times 10^{25}$ y, in order to cover the ellipse segment left out by the current EXO \oplus KL-Zen bound. If correlations were neglected, the ellipse would not be tilted, and this requirement would (incorrectly) become less stringent.

We remark that the results shown in this section are based on the same NME and covariances as in [1]. In general, it would be useful to extend the theoretical covariance analysis in further directions, including: (i) updated and improved QRPA calculations; (ii) other candidate $0\nu\beta\beta$ nuclei not considered in [1]; (iii) theoretical NME approaches different from the QRPA (see also the Appendix); (iv) nonstandard decay mechanisms. From the experimental viewpoint, we have emphasized the importance of publishing the probability distribution of the half life for each nucleus. All these refinements will become increasingly important in the next few years, since a number of $0\nu\beta\beta$ experiments will provide highly significant data, which must be eventually combined in proper theoretical and statistical frameworks.

III. SUMMARY

In the previous work [1] we presented estimates of NME and their covariances for a set of candidate $0\nu\beta\beta$ nuclei, within the QRPA theoretical framework. Such estimates still provide a fair representation of the spread of NME calculations (see the Appendix). In this context, we have compared herein the claimed ^{76}Ge signal from [2, 3] with negative results from other experiments, including the recent ^{136}Xe limits placed by EXO-200 [4] and KamLAND-Zen [5]. We have worked out favored and disfavored ranges at 90% C.L. for each nucleus and for couples of nuclei, in

terms of either half lives T_j (Figs. 1, 3 and 4) or of $m_{\beta\beta}$ (Fig. 2). In particular, we find that, in order to close the region currently allowed at 90% C.L. by the ^{76}Ge claim and by the ^{136}Xe limit, one should cover either the range $T(^{76}\text{Ge}) \simeq 2.0\text{--}2.9 \times 10^{25}$ y or the range $T(^{136}\text{Xe}) \simeq 3.4\text{--}4.3 \times 10^{25}$ y; alternatively, using a third nucleus such as ^{130}Te , one should cover the range $T(^{130}\text{Te}) \simeq 0.7\text{--}1.1 \times 10^{25}$ y (see also Fig. 4 for other nuclei). We remark that the theoretical NME covariances play a relevant role in these or similar estimates: their study should thus be further pursued, not only within the QRPA [1], but also within other approaches as well as for nonstandard $0\nu\beta\beta$ mechanisms. We have also emphasized that experimental results should be given in terms of likelihood functions for the decay half-life (rather than in terms of bounds at a fixed C.L.), in order to allow a proper combination of the experimental and theoretical uncertainties, which are equally important to derive constraints on $0\nu\beta\beta$ parameters.

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APPENDIX

In the Appendix of Ref. [1], we clarified the role of different conventions about the phase space factors G_i and the nuclear matrix elements $|M'_i|$. We also compared our QRPA estimates for the NME logarithms η_i with those calculated independently in [11] (within the QRPA) and in [12, 13] (within the shell model, SM), which were all encompassed by our $\pm 3\sigma$ ranges ($\eta_i^0 \pm 3\sigma_i$). In this Appendix, we show that such ranges also embrace more recent NME calculations performed via the Energy Density Functional method (EDF) [14], the microscopic Interacting Boson Model (IBM-2) [15], and the projected Hartree-Fock-Bogoliubov method (PHFB) [16], as well as the Renormalized QRPA (RQRPA) approach [17]. The last calculation is particularly relevant with respect to [1], since it embeds the recent experimental determination of the $2\nu 2\beta$ half life in ^{136}Xe [18, 19] (not available at the time of [1]), which fixes the so-called g_{pp} parameter of the QRPA.

Different NME calculations may use slightly different phase space factors (see the recent detailed evaluation in [20]), different values of the nuclear radius parameter r_0 , and different conventions. In order to make a homogeneous comparison with the η values and the conventions of [1], we use the fact that the papers [14–17] contain tables of estimated half lives (τ_i) at fixed values of the effective Majorana mass (μ); then, by adopting the same phase space factor (γ_i) reported in [1], the NME values (η_i) can be calculated via Eq. (3) and can be directly compared with those in [1]. For this purpose, we use the half lives in Table I of [14], in Table III of [15], in Table IV of [16] (central values), and in Table I of [17] (central values). From [17] we select two options, RQRPA with Jastrow short-range correlations, and QRPA with CD-Bonn potential, which generally provide the lowest and highest NME values, respectively [17]. We note that the papers [16, 17] also report estimated NME uncertainties, but not their covariance matrix; in any case, the errors estimated in [1] are generally more conservative than those in [16, 17].

Table II reports the effective η_i of such recent EDF, IBM-2, PHFB, and (R)QRPA calculations, to be compared with the $\pm 3\sigma$ ranges from [1] in the last two rows. Such ranges largely embrace all the above values. Actually, almost all the NME in Table II are contained in the 90% C.L. ranges ($\eta_i^0 \pm 1.64\sigma_i$, not shown). We conclude that Table I of [1] still provide a reasonable and conservative evaluation of the NME η_i and of their variances $\sigma_i^2 = \text{var}(\eta_i)$.

TABLE II: Estimates of $\eta_i = \log_{10} |M'_i|$ for each nucleus, as derived from the recent EDF [14], IBM-2 [15], and PHFB [16] calculations after appropriate rescaling, in order to match the conventions used in [1]. The estimates of [16] refer only to a subset of nuclei. Also shown are the η_i central values for two widely different (R)QRPA models recently reported in [17]. The corresponding values of the adopted effective axial coupling g_A are also reported. The last two rows report the upper and lower ends of our 3σ range $\eta_i^0 \pm 3\sigma_i$ as taken from [1], which largely encompass the above η_i values.

Ref.	Model	g_A	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
[14]	EDF	1.25×0.74	0.617	0.577	0.707	0.663	0.629	0.570	0.665	0.571
[15]	IBM-2	1.269	0.721	0.622	0.390	0.564	0.421	0.616	0.562	0.467
[16]	PHFB	1.254			0.474	0.813		0.586	0.623	
[16]	PHFB	1.0			0.317	0.658		0.433	0.470	
[17]	RQRPA (Jastrow)	1.0	0.535	0.460	0.045	0.365	0.289	0.401	0.291	0.160
[17]	QRPA (CD-Bonn)	1.25	0.797	0.748	0.316	0.717	0.597	0.736	0.689	0.468
[1]	Lower limit of our 3σ range		0.269	0.166	−0.703	0.017	−0.046	0.072	0.024	−0.307
[1]	Upper limit of our 3σ range		1.001	0.976	0.779	0.989	0.854	0.996	0.972	0.815

Concerning the NME covariances, $\text{cov}(\eta_i, \eta_j) = \rho_{ij}\sigma_i\sigma_j$, no comparison is possible within the current literature, since they have been evaluated only in [1]. Here we just remind their crucial role, by deriving the last three equations in Sec. II A. Since the experimental error s_i ($i = {}^{76}\text{Ge}$) in Eq. (10) is independent from any theoretical error σ_j , it is $\text{cov}(\tau_i, \eta_j) = 0$; in addition, phase space uncertainties are currently negligible, $\text{var}(\gamma_j) \simeq 0$. Thus, in the nontrivial case $j \neq i$, propagation of errors in Eq. (12) gives

$$\begin{aligned} s_j^2 &\equiv \text{var}(\tau_j) \\ &= \text{var}(\tau_i) + 4[\text{var}(\eta_j) + \text{var}(\eta_i) - 2\text{cov}(\eta_i, \eta_j)] \\ &= s_i^2 + 4(\sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j) \end{aligned} \quad (16)$$

and

$$\begin{aligned} r_{ij}s_i s_j &\equiv \text{cov}(\tau_i, \tau_j) \\ &= \text{cov}(\tau_i, \tau_i) \\ &= s_i^2, \end{aligned} \quad (17)$$

as reported in Eqs. (13) and (14). Similarly, for $j \neq i$ and $k \neq i$ (where $i = {}^{76}\text{Ge}$), it is

$$\begin{aligned} r_{jk}s_j s_k &\equiv \text{cov}(\tau_j, \tau_k) \\ &= \text{cov}(\tau_i, \tau_i) + 4\text{cov}(\eta_i, \eta_i) - 4\text{cov}(\eta_i, \eta_k) - 4\text{cov}(\eta_i, \eta_j) + 4\text{cov}(\eta_j, \eta_k) \\ &= s_i^2 + 4\sigma_i^2 - 4\rho_{ij}\sigma_i\sigma_j - 4\rho_{ik}\sigma_i\sigma_k + 4\rho_{jk}\sigma_j\sigma_k \\ &= \frac{1}{2}(s_j^2 + s_k^2) - 2(\sigma_j^2 + \sigma_k^2 - 2\rho_{jk}\sigma_j\sigma_k), \end{aligned} \quad (18)$$

as reported in Eq. (15). The relevance of NME covariances clearly emerges in the above equations.

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